New Analytical Results for *Löfgren C* Tonearm Alignment

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This author's recent paper on the zeros of the tracking error for various Löfgren alignments showed that the formula originally derived in 1941 for tracking angle zeros in the case of the *Löfgren A* alignment method ("minimax" optimization of distortions) provides accurate results in practice, but the approximate formula often used for the *Löfgren C* alignment (Least Mean Squares optimization) does not appear to work as well. The zero tracking error radii were found to be in error by up to 0.6 mm, causing practically all protractors for *Löfgren C* alignment to be slightly miscalibrated. This paper investigates the *Löfgren C* case analytically and presents some new formulae for the optimum offset angle, overhang, and zero tracking error radii, which match the numeric optimization results very well.

0 INTRODUCTION

Horizontal tracking error (HTE), also known as lateral tracking error, is one of the main sources of distortion in vinyl disk reproduction. (Others are vertical tracking error, elastic and non-elastic deformation of the vinyl, and tracing errors that come from the difference between the reproducing stylus shape and the shape of the cutting chisel used to make the original acetate lacquer master.) Since most of the original theoretical work on HTE was completed before World War II, comparatively few papers on the subject were published in this journal, which was launched in 1953. Given the renewed interest in vinyl disks in recent years [1], an introductory description of the problem is provided and followed by some new results.

1 HORIZONTAL TRACKING ERROR

HTE is a consequence of the fact that in most disk-cutting lathes, the stylus that engraves the acetate moves strictly tangentially to the groove during the recording of the master, along a radius of the disk. With the usual pivoted tonearms, however, the reproducing stylus moves in an arc so that the cartridge can be at right angles to a radius at no more than two groove positions over the whole disk radius. HTE angle δ is then the angle between the axis of the pickup cartridge[1] and the line perpendicular to the radius

at the point where the stylus touches the disk, as depicted in Fig. 1. By using a cosine theorem, the HTE angle δ in a grove at radius r can be calculated as

$$\delta = \arcsin\left[\frac{r^2 + L^2 - (L-H)^2}{2rL}\right] - \beta, \qquad (1)$$

where L is the effective tonearm length (stylus–to–pivot-point distance), and β and H are the tonearm's offset angle and overhang respectively, as shown in Fig. 1.

In 1924 Wilson [2] provided formulae for the optimum overhang and offset angle to minimize the HTE angle δ. It was a "minimax" optimization, which ensured that the maximum values of δ along the record were as small as possible.

2 TRACKING DISTORTION

If the magnetic cartridge "reading" the groove is slanted by angle δ to the groove, as in Fig. 2, at some point x along the groove the stylus displacement will be $\overline{xb}$ instead of the correct value $\overline{xa}$, i.e., instead of displacement from the groove axis to the point marked with a triangle, reproduced displacement will be from the axis to the black circular dot ($\overline{xc}$). An atypically large HTE angle is used in Fig. 2 so that the effect is more pronounced and easier to observe.

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[1]A more correct definition uses the "vertical stylus plane" instead of the "cartridge axis." It is a vertical plane encompassing

the stylus cantilever, and the HTE is the angle between the line perpendicular to the radius at the point of contact and the line at the intersection of the "vertical stylus plane" with the horizontal disk plane.
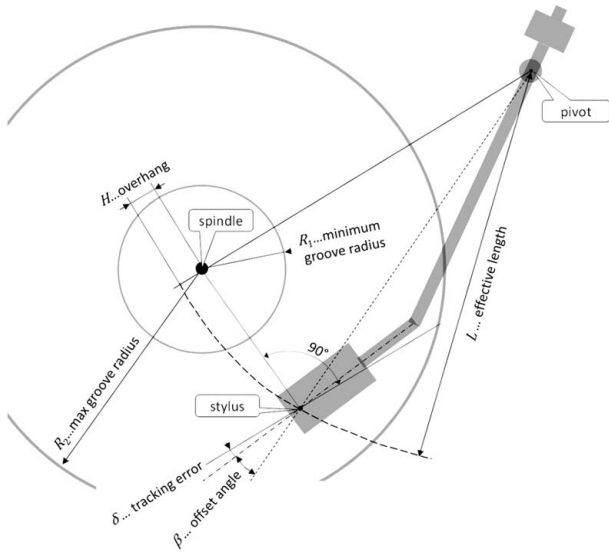
Fig. 1. Tonearm geometry.

If the recorded waveform is $f_i(x)$, simple trigonometry shows that the output waveform $f_o(x)$ will be a scaled version of that waveform at some distance $x - \xi$.

$$f_o\ (\mathrm{x}) = \frac{f_i\ (\mathrm{x} - \xi)}{cos\ (\delta)}\ , \tag{2}$$

which can be found by solving the equation

$$\xi\ =\ f_i\ (\mathrm{x} - \xi)\ tan\ (\delta)\,. \tag{3}$$

If $f_i(x)$ corresponds to the sinusoidal recorded tone, nowadays one would probably solve Eq. (3) numerically for ξ in 512 or 1,024 equally spaced points along x, find $f_o(x)$ in all those points by solving Eq. (2) numerically (which is how the distorted curve at the bottom of Fig. 2 was created), and then perform a fast Fourier transform.

Without computers, however, in 1938 Swedish engineer Erik Löfgren realized that—although the explicit closed-form expression for the waveform $f_o(x)$ does not exist in the case of a sine wave—the Fourier analysis of Eq. (2) can still be performed analytically. For a sinusoidal tone of amplitude A and wavelength λ recorded along the groove, he showed that the amplitude A_n of the nth harmonic of the output waveform $f_o(x)$ will be given by [3],

$$A_n = \frac{A}{\cos\ (\delta)}\ \frac{2}{n\varepsilon}\ J_n\ (n\varepsilon)\,. \tag{4}$$

In Eq. (4), $J_n(\cdot)$ stands for the Bessel function of the first kind, n^{th} order, and parameter ε is given by

$$\varepsilon\ =\ \frac{2\pi A}{\lambda}\ \tan\ (\delta)\,. \tag{5}$$

The recorded wavelength λ in the groove corresponds to the distance that stylus traverses during one period of the recorded tone at frequency f, i.e.,

$$\lambda\ =\ r\frac{\Theta}{f}, \tag{6}$$

where Θ is angular disk revolution speed. For Long-Play (LP) records with 331/3 rpm, $\Theta = 2\pi \cdot 331/3/60$.
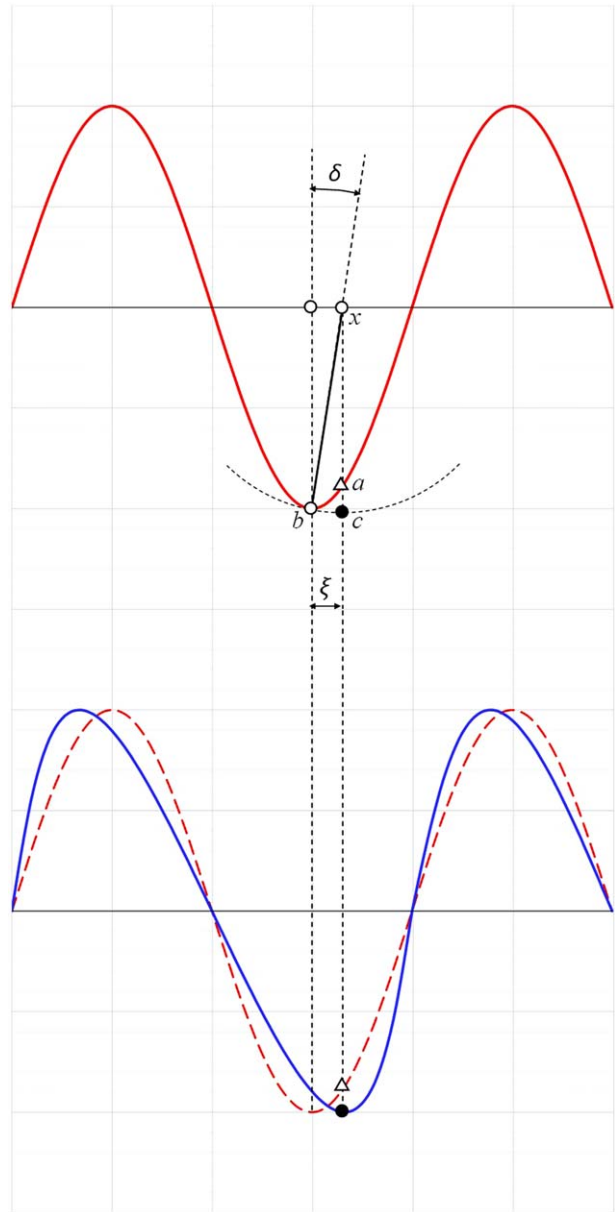

Fig. 2. Tracking distortion waveforms for $\delta = 10°$.

The output voltage of the modern magnetic cartridge is proportional to the stylus velocity rather than its displacement. The recorded velocity—being a derivative of displacement in the time domain—is a product of the recorded amplitude A and angular frequency $2\pi f$ of the recorded tone, so Eq. (5) can be expressed in terms of the peak recorded velocity, which is denoted by v_{max}, as

$$\varepsilon\ =\ \frac{v_{max}}{\Theta r}\ \tan\ (\delta)\,. \tag{7}$$

After Löfgren reworked Eq. (4) to reflect the velocities and expanded the Bessel functions into the Taylor series, he derived the approximate expressions for the first few harmonic components as

$$V_1 = \frac{v_{max}}{\cos\ (\delta)}\ \left(1 - \frac{\varepsilon^2}{8} + \dots\right),$$

$$V_2 = \frac{v_{max}}{\cos(\delta)} \, \varepsilon \left(1 - \frac{\varepsilon^2}{3} + \ldots\right),$$

$$V_2 = \frac{v_{max}}{\cos(\delta)} \, \frac{9}{8}\varepsilon^2 \left(1 - \frac{9\varepsilon^2}{16} + \ldots\right)$$

$$\ldots \tag{8}$$

From this Löfgren correctly deduced that since the ε in practice will be of the order of 0.01, the distortion will be almost exclusively in the second harmonic and that the total harmonic distortion (THD) for almost all practical purposes can be approximated by a value $k(r)$ given by

$$k(r) \approx |\varepsilon| = \frac{v_{max}}{\Theta} \left|\frac{\tan(\delta)}{r}\right| = \frac{v_{max}}{\Theta} |wte(r)|, \tag{9}$$

where $wte(r)$—the tangent of the tracking error divided by the radius—became known as the *weighted tracking error*.

Numerical evaluation via Eq. (3) and fast Fourier transform show that these approximate results are very accurate. For instance, if the parameters are chosen so that $k(r)$ corresponds to a THD = 2% (before the RIAA equalization[2] on replay), the real distortion due to the first harmonic is at 1.9999%, second at 0.0300%, and third at 0.0004%; the real numerically calculated THD is at 2.00013%.

This achievement by Löfgren is especially remarkable since the only other prior result for tracking distortion [4] was obtained by drawing the distorted curves apparently manually on paper, so that one wavelength was 40 cm long, and then using an analog computer[3] to trace them and calculate the harmonic content numerically for three different cases only—and the results were wrong [3].

Even more amazing is that Löfgren managed to calculate the intermodulation distortion components: first-order artifacts (at $f_1 \pm f_2$) were also proportional to ε, and the higher-order artifacts (e.g., $2f_1 \pm f_2$) were proportional to ε^2. This clearly showed that in order to minimize the distortions, it was the parameter $|wte(r)|$ that needed to be optimized.

3 *LÖFGREN A* DISTORTION MINIMIZATION

After calculating the distortions, Löfgren faced an insurmountable task to analytically optimize the function $wte(r)$, which—from Eqs. (7) and (1)—turns into

$$wte(r) = \frac{\tan\left[arcsin\left[\frac{r^2+2LH-H^2}{2rL}\right] - \beta\right]}{r}. \tag{10}$$

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[2]Variable slope of the RIAA curve reduces the second harmonic by a factor of between about 2 (if they fall in the high) and 1.2 (if in the low end of the audio spectrum), with a local minimum of 1.3 near 750 Hz and local maximum of about 1.8 near 120 Hz.

[3]"Henrici-Coradi Harmonic Analyzer" at the Eastman Kodak Company Research Laboratories. The author's search for more information about this early computing device did not give any results.

Being an expert mathematician, he managed to derive a much simpler approximation for Eq. (10) that proved tractable:

$$wte(r) \approx \frac{\frac{r^2+2LH-H^2}{2rL} - \sin(\beta)}{r \cdot \cos(\beta)}. \tag{11}$$

The accuracy of this approximation is outstanding once again, as can be verified from Fig. 3 for a set of quite typical system parameters—the two curves are practically indistinguishable.

The first optimization method Löfgren proposed in [3] was once again the "minimax" approach, but—unlike Wilson—he minimized the maximum *distortion* along the radius, not the maximum value of the HTE itself. The shape of the curve in Fig. 3 suggested that the maximums of the $|wte(r)|$ will appear in the groove with the minimum radius (R_1), with maximum radius (R_2) on the disk, and at a radius R_m between where the $wte(r)$ function has the minimum. The value of R_m can be found by evaluating a derivative of Eq. (11) and finding its zero, after which the condition

$$wte(R_1) = -wte(R_m) = wte(R_2) \tag{12}$$

gives two equations for optimum β and H. Denoting the optimum values for *Löfgren A* minimization with the superscript "A," the solutions are[4]

$$\beta^A = \arcsin\left[\frac{4R_1R_2(R_1 + R_2)}{L\left(R_1^2 + 6R_1R_2 + R_2^2\right)}\right], \tag{13}$$

$$H^A = L - \sqrt{L^2 - \frac{8R_1^2R_2^2}{R_1^2 + 6R_1R_2 + R_2^2}}. \tag{14}$$

Comparison of the results from Eqs. (13) and (14) with the exact values (named "perfect" results per Dennes [5]) obtained by minimizing the maximums of the $|wte(r)|$ numerically shows that they give accurate results for probably all practical purposes. The relative errors are around 0.1%, and the absolute errors are within 0.2 mm for overhang H and $0.2°$ for offset angle β for the tonearms of length above 228.6 mm (9 in), which is traditionally a minimum length for high-quality pivoted tonearms [6]. Errors are even smaller when the tonearm is longer.

Another useful result for *Löfgren A* alignment was derived in 1941 by Baerwald [7], who showed that the HTE zeros will be located at radii R_{01} and R_{02} given by

$$R_{01/02}^A = \frac{2R_1R_2}{\left(1 \pm \frac{1}{\sqrt{2}}\right)R_2 + \left(1 \mp \frac{1}{\sqrt{2}}\right)R_1}, \tag{15}$$

which depend on the disk first (R_1) and last (R_2) groove radii but are independent of the tonearm length. This proved to be useful because mounting templates (usually called "protractors") could be used to optimize the tonearm pivot position and offset angle (if adjustable) for a tonearm of

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[4]In [5], pp. S2-4–S2-5, the original Löfgren's formulae were parametric, but presented here are the versions that show explicitly that the optimum tonearm parameters depend only on the inner (R_1) and outer (R_2) disk radii and tonearm length (L).
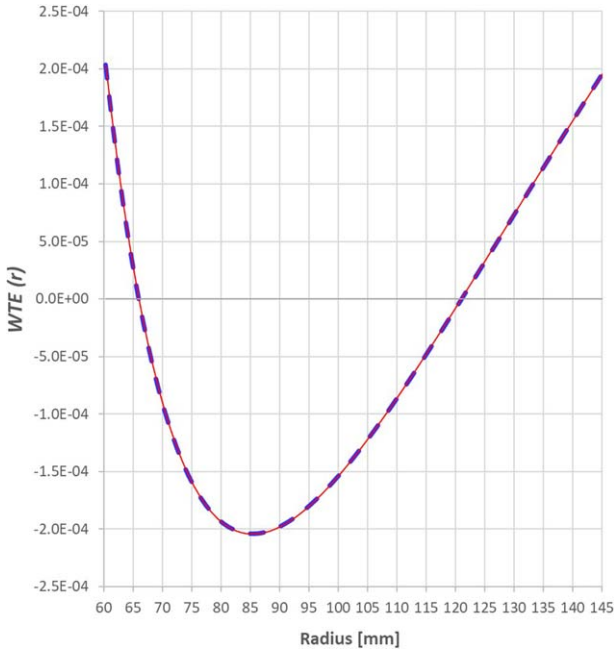
Fig. 3. Exact weighted tracking error, $wte(r)$ along the LP disk radius from Eq. (10) (thin solid line) and approximation from Eq. (11) (thicker dashed line).

Fig. 4. Protractor for tonearm adjustment.

any length by ensuring that the HTE is zero at these two radii. One example of such a protractor is shown in Fig. 4.

4 *LÖFGREN B* AND "APPROXIMATE" *LÖFGREN C* ALIGNMENT

After solving the "minimax" problem, Löfgren in his 1938 paper [3] also considered the Least Mean Squares (LMS) criterion, the optimization method probably most often used for the problems of this kind. He basically looked

into optimizing the mean-square weighted tracking error along the radius of the disk, i.e.,

$$\overline{wte_{LMS}^2}(L, H, \beta) = \int_{R_1}^{R_2} [wte(r)]^2 \, dr, \qquad (16)$$

with the idea that useful results might be obtained by considering all distortion values along the radius, not just the three maximum ones, and in the process giving more weight to the larger distortion values than to the smaller ones.

By using the approximation Eq. (12), Löfgren was able to calculate the integral in Eq. (16) as

$$
\overline{wte_{LMS}^2}(L, H, \beta)
$$
$$
= \frac{\frac{R_2 - R_1}{4} - p \ln\left(\frac{R_2}{R_1}\right) + \left(\frac{a^2}{2} + p^2\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}{(L^2 - p^2)(R_2 - R_1)}
$$
$$
- \frac{\frac{pa^2}{2}\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right) - \frac{a^4}{12}\left(\frac{1}{R_1^3} - \frac{1}{R_2^3}\right)}{\left(L^2 - p^2\right)(R_2 - R_1)}, \qquad (17)
$$

where he introduced two auxiliary variables to simplify the notation, namely

$$p = L \sin(\beta), \qquad (18)$$

$$a^2 = 2LH - H^2. \qquad (19)$$

The optimum tonearm parameters can now be determined by finding partial derivatives of $\overline{wte_{LMS}^2}(L, H, \beta)$ versus a^2 and p, and by solving a system of equations

$$\frac{\partial \overline{w_{LMS}^2}(L, H, \beta)}{\partial (a^2)} = 0, \qquad (20)$$

$$\frac{\partial \overline{w_{LMS}^2}(L, H, \beta)}{\partial p} = 0 \qquad (21)$$

for a^2 and p. The optimum overhang H and offset angle β can then be obtained from Eqs. (18) and (19).

Given the complexity of Eq. (17), this task was obviously formidable, but Löfgren managed to solve Eq. (20). His result was

$$H^B = L - \sqrt{L^2 - \frac{3R_1 R_2 [L \sin(\beta)(R_1 + R_2) - R_1 R_2]}{R_1^2 + R_1 R_2 + R_2^2}}. \qquad (22)$$

For the reasons that are not completely clear from the present perspective, he preferred to offer yet another approximation

$$H^B \approx \frac{3R_1 R_2 [L \sin(\beta)(R_1 + R_2) - R_1 R_2]}{2L(R_1^2 + R_1 R_2 + R_2^2)}. \qquad (23)$$

Unlike most of his other approximations, Eq. (23) seems only marginally simpler than Eq. (22) but creates a relative error of around 3% under typical systems parameters.

Löfgren did not report anything about solving Eq. (21). Instead, he proposed that Eq. (23) could be used to optimize the overhang (and thus the pivot location) for a certain given offset angle β. That might look a bit strange nowadays, but it was not so in 1938, when the tonearm parameters were often far from any optimum because the manufacturers were

unfamiliar with the principles of any optimum designs and offset angle β could not be adjusted. It can be useful even now, since many modern tonearms are still produced with offset angles that do not conform to the optimum values under any mathematically justified alignment method and "shells" that carry the cartridge at the end of the tonearms often allow for a limited offset angle adjustment range only, if any.

The superscript "*B*" in Eqs. (22) and (23) comes from the fact that this alignment, constrained by the given fixed offset angle β, was named "*Löfgren B*" in the seminal paper on tracking errors by Dennes, which is a must-read for anybody interested in this subject [5]. Dennes then coined the term *Löfgren C* for the unconstrained LMS alignment optimization and devised a numerical procedure for finding both optimum parameters using the ubiquitous "*Microsoft Excel*" and its "*Solver*" add-in. It should be noted again that Löfgren himself did not offer any solutions for the *Löfgren C* optimization method, nor did he introduce any names for the three distinct alignment methods he discussed in his 1938 paper [3].

An approximate solution for the *Löfgren C* alignment seems to have been proposed for the first time by this author

tonearms based on both *Löfgren A* and *Löfgren C* methods [9]. Versions of these formulae, developed independently by Rampelmann [10] and Elison [11], appeared almost simultaneously in English in 2000 but only in the parametric form (using the linear offset term $L \cdot \sin(\beta)$ as a parameter). In 2001 Kearns published a justification for the zero radii constancy and some novel results for the relationship between the zeros and other pertinent tonearm parameters [12,13].

Eq. (26) was routinely used to calculate the zeros for several protractors for optimization following the *Löfgren C* alignment, which is the preferred alternative to the better known *Löfgren A* method for some audiophiles.[5]

5 "EXACT" *LÖFGREN C* ALIGNMENT[6]

This author tried to solve Eq. (21) manually a number of times in the 1980s, but the computational complexity involved in finding the result seemed overwhelming. The software tools for symbolic mathematical calculations since then changed this situation drastically, and the following formula for the "Exact" linear offset p was finally derived, as shown in Eq. (27):

$$p=\frac{(L^2 + R_1 R_2)(R_2 - R_1)^3 - \sqrt{(R_2-R_1)^6(L^2+R_1R_2)^2 - 4L^2R_1^2R_2^2[2(R^2_1+R_1R_2+R_2^2)\,log\,(\frac{R_2}{R_1}) - 3(R_2^2 - R_1^2)]^2}}{4R_1R_2(R_1^2 + R_1R_2 + R_2^2)log\,(\frac{R_2}{R_1}) - 6R_1R_2(R_2^2 - R_1^2)}.$$

(27)

in 1981 in his B.Sc. thesis [8]. After solving Eqs. (20) and (21) numerically on a mainframe computer for a series of tonearm lengths of practical interest, he confirmed that the optimum offset angle under the unconstrained *Löfgren C* method is very close to the offset angle β^A under the *Löfgren A* alignment conditions. He then proposed that the β^A, coupled with the more exact formula Eq. (22) for the overhang, can be used for the *Löfgren C* case and obtained the following results for the optimum tonearm parameters:

$$\beta^C(= \beta^A) \approx \arcsin\left[\frac{4R_1R_2(R_1 + R_2)}{L\left(R_1^2 + 6R_1R_2 + R_2^2\right)}\right], \quad (24)$$

$$H^C \approx L - \sqrt{L^2 - \frac{9R_1^4R_2^2 + 6R_1^3R_2^3 + 9R_2^4}{R_1^4 + 7R_1^3R_2 + 8R_1^2R_2^2 + 7R_1R_2^3 + R_2^4}}.$$

(25)

Further calculations based on Eqs. (24) and (25) gave the zero tracking error radii for *Löfgren C* alignment as

$$R_{01/02}^{C*} = \frac{R_1R_2}{R_1^2 + 6R_1R_2 + R_2^2}\left[4(R_1 + R_2)\right.$$
$$\left.\mp \sqrt{\frac{7R_1^4 - 12R_1^3R_2 + 10R_1^2R_2^2 - 12R_1R_2^3 + 7R_2^4}{R_1^2 + R_1R_2 + R_2^2}}\right].$$

(26)

While Eqs. (24) and (25) were never made public, Eq. (26) appeared in a patent for a protractor for adjusting the

From Eqs. (18), (19), and (22), it is now possible to obtain the formulae for the "Exact" overhang H^C, offset angle β^C, and zeros of the tracking error $R_{01/02}^C$ in terms of the tonearm length L and extreme groove radii R_1 and R_2, but the formulae are extremely complex,[7] so the author chose to give much simpler parametric expressions. Note that these results are "Exact" in the sense that the author was still using the accurate approximation Eq. (11) instead of the true weighted tracking error from Eq. (10), but the errors are expected to be small. If the auxiliary parameter.

$$a^2 = \frac{3R_1R_2[p(R_1 + R_2) - R_1R_2]}{R_1^2 + R_1R_2 + R_2^2} \quad (28)$$

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[5]*Löfgren A* is sometimes called *Löfgren/Baerwald* or *Baerwald* alignment, which is not justified—except for the null radii formula from Eq. (15), Baerwald did little but copy Löfgren's results without giving him proper credit ([5], pp. S1-54–S1-57).

[6]As with the *Löfgren A/Baerwald* naming confusion, the *Löfgren C* alignment method is presently often known as *Löfgren B* among audiophiles. This is wrong largely because of the misunderstanding of what Löfgren actually accomplished prior to his 1938 paper [3] being translated into English in 2008.

[7]Each of the three non-parametric formulae corresponding to Eqs. (29)–(31) requires two or three lines across both columns to show and about half a page of dense math formulae together. Upon request, the author will provide them either in the "Mathematica" notebook format or PDF version of that file.

Table 1. Optimum overhang H^C and offset angle β^C: exact numeric results and "Exact" results from Eqs. (29) and (30), with the associated errors (Δ).

Length L (mm)	220	240	260	280	300
"Perfect" H^c (mm)	19.28	17.56	16.12	14.91	13.87
"Perfect" β^c (°)	24.99	22.80	20.96	19.41	18.07
"Exact" H^C (mm)	19.26	17.54	16.11	14.91	13.87
"Exact" β^C (°)	24.97	22.78	20.95	19.40	18.07
ΔH^C (mm)	0.02	0.01	0.01	0.01	0.01
$\Delta \beta^C$ (°)	0.02	0.01	0.01	0.01	0.01

Table 2. Optimum overhang H^C and offset angle β^C: exact numeric results and approximate results from Eqs. (24) and (25), with the associated errors (Δ).

Length L (mm)	220	240	260	280	300
"Perfect" H^c (mm)	19.28	17.56	16.12	14.91	13.87
"Perfect" β^c (°)	24.99	22.80	20.96	19.41	18.07
"Approx." H^C (mm)	19.49	17.73	16.27	15.04	13.99
"Approx." β^C (°)	25.13	22.91	21.06	19.50	18.15
ΔH^{C*} (mm)	−0.21	−0.17	−0.15	−0.13	−0.11
$\Delta \beta^{C*}$ (°)	−0.14	−0.12	−0.10	−0.09	−0.08

is introduced, then the offset angle and overhead are

$$\beta^C = \arcsin\left(\frac{p}{L}\right), \tag{29}$$

$$H^C = L - \sqrt{L^2 - a^2}. \tag{30}$$

The formula for the zeros of the HTE then becomes

$$R_{01/02}^C = p \mp \sqrt{p^2 - a^2}. \tag{31}$$

It is worth noticing that these zeros are not constant for given values of extreme groove radii R_1 and R_2; they depend on the tonearm length L because p from Eq. (27) depends on L [although a^2 from Eq. (28) does not].

How do these "Exact" formulae for the *Löfgren C* case compare with the "Perfect" results obtained by solving numerically for the minimums of $\overline{w_{LMS}^2}(L, H, \beta)$ using the accurate version of the $wte(r)$ from Eq. (10) rather than the approximation from Eq. (11)? The methodology originally devised by Dennes ([5], pp. S1-15–S1-24) permits one to calculate the "Perfect" (truly exact) values numerically again.

For the first and last groove radii $R_1 = 60.325$ mm and $R_2 = 146.05$ mm, as standardized by the International Electrotechnical Commission, a comparison of the "Perfect" results with the "Exact" results for optimum overhang and offset angle from Eqs. (29) and (30) is shown in Table 1, along with the corresponding errors. An analogous comparison of the "Perfect" results with the previously available "Approximate" results obtained via Eqs. (24) and (25) is shown in Table 2. It can be verified that the "Exact" results are considerably more accurate, giving errors that are smaller by about an order of magnitude. Accuracy is actually very similar to what the *Löfgren A* formulae Eqs. (13) and (14) give, which should not be surprising since the only approximation involved in Eqs. (13) and (14) and

Table 3. "Perfect" zero radii obtained numerically, by the "Approximate" formula Eq. (26) and "Exact" formula Eq. (31), and the associated errors for *Löfgren C* alignment.

Length L (mm)	220	240	260	280	300
"Perfect" R_{01}^C (mm)	69.97	70.01	70.04	70.05	70.07
"Perfect" R_{02}^C (mm)	115.91	115.97	116.00	116.03	116.05
"Approx." R_{01}^C (mm)	70.29	70.29	70.29	70.29	70.29
"Approx." R_{02}^C (mm)	116.60	116.60	116.60	116.60	116.60
"Exact" R_{01}^C (mm)	70.03	70.05	70.07	70.09	70.10
"Exact" R_{02}^C (mm)	115.73	115.82	115.88	115.93	115.97
"Approx." ΔR_{01}^C (mm)	−0.310	−0.274	−0.248	−0.228	−0.213
"Approx." ΔR_{02}^C (mm)	−0.697	−0.645	−0.606	−0.576	−0.552
"Exact" ΔR_{01}^C (mm)	−0.050	−0.040	−0.034	−0.028	−0.024
"Exact" ΔR_{02}^C (mm)	0.175	0.141	0.116	0.099	0.085

Eqs. (29) and (30), is the very accurate approximation for $wte(r)$ from Eq. (11).

It is further interesting to compare how the "Exact" and "Approximate" *Löfgren C* formulae compare in terms of the zero tracking error radii with the numerically obtained "Perfect" results. Calculation from the "Approximate" Eq. (26) gives the zero radii at 70.285 and 116.604 mm, independent of the tonearm length, while both the "Perfect" and "Exact" results for zeros have a slight variation, mostly within 0.15 mm with tonearm lengths likely to be encountered in practice.

Just like in the case of overhang and offset angle, the new "Exact" formula gives errors that are considerably smaller than the previous "Approximate" ones for *Löfgren C* alignment. While it can be argued that the improvements by less than 0.2 mm and 0.15° are close to the manufacturing tolerances and are thus probably not very significant, the accuracy improvement for the larger zero tracking radii is up to 0.6 mm, which is likely larger than the typical accuracy of the tonearm adjustment procedures when performed manually via the protractors, and certainly well above what can be achieved in industrial production of turntables that come equipped with the tonearms from the factories.

Furthermore from Table 3 it can be seen that the zero radii of 70.02 and 115.98 mm would be convenient zero points for use in protractors. They will be within 0.07 mm of the true zero points for all high-end tonearms of practical interest.

It is probably also interesting to compare the actual distortions under *Löfgren A* and *Löfgren C* alignment methods. The corresponding results for a typical tonearm are shown in Fig. 5. It can be seen that the distortions can reach considerable values even with the relatively modest recorded velocity of 10 cm/s—for larger velocities, the distortion would increase proportionally, i.e., at 40 cm/s maximums would exceed 2%.[8]

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[8]The maximum possible recorded velocity on an LP is a subject of considerable debate when the dynamic ranges of vinyl versus digital are discussed. The magnetic cartridges were able to track velocities approaching 50 cm/s about half a century ago, and there were reports that the peak velocity of 105 cm/s was measured on an actual commercial record ([16], p. 72) issued in 1963.
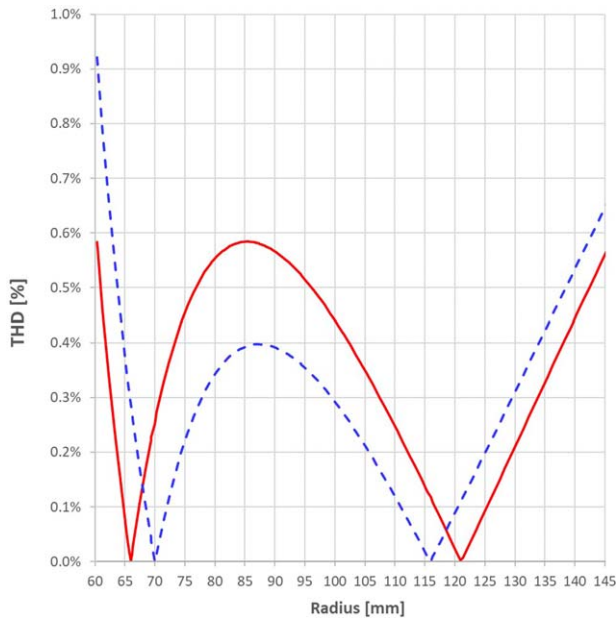
Fig. 5. Total harmonic distortion (THD) along the radius of an LP record at $33\frac{1}{3}$ rpm with recorded peak velocity of 10 cm/s; $L=$ 250 mm tonearm. The solid line is for *Löfgren A*, and the dashed line is for *Löfgren C* alignment.

Revived interest in *Löfgren C* alignment among audiophiles since about the turn of this century could be in part because of the fact that the dominant disk speed in 1938, when *Löfgren A* method was devised, was 78 rpm, which according to Eq. (9) gives $78/33\frac{1}{3} = 2.34$ times lower distortion numbers than the present speed of $33\frac{1}{3}$ rpm. With peak velocities of 10 cm/s, the corresponding worst-case distortion at that time would have been about 0.25%. Given that recorded velocities larger than that value were not likely in the pre–World War II era and that maximum distortion of 0.25% was at those times considered to be inaudible or perhaps barely audible, ensuring that the worst-case distortion values would stay in that range was a rather sensible approach.

With lower record speed and larger recorded velocities, audiophiles nowadays might be able to hear distortions not just at the peaks but also over a considerable portion of the disk, and the method that reduces them over a larger area might be preferable to them, in spite of a small increase in distortions in the first 5 mm or so on the disk and fairly large increase in about the last 5 mm.

Increased distortion in the last 5 mm or so on the record is a serious concern, but it is alleviated at least to some extent by the fact that very few records actually have any music material recorded up to the minimum radius limit. In 1980 [8] this author found that 50% of the records did not have recorded content under a 65-mm radius, and more recent measurements on a much larger sample size indicate that since the 1990s that percentage might be well into single digits ([17], p. 5).

6 CONCLUSIONS

This paper presented an overview of the tonearm designs to minimize horizontal tracking error and reported some new results for the 80-year-old problem related to the *Löfgren C* alignment method, which utilizes LMS optimization criterion. In particular, new formulae were derived for calculating the optimum overhang and offset angle as a function of the tonearm length, which are almost an order of magnitude more accurate than the existing approximate ones. They give negligible errors (within 0.02 mm for overhang and $0.02°$ for offset angle) from the truly optimal values obtained by numeric minimization on a computer.

A new formula was also derived for the zeros of the tracking angle that can be useful for design of protractors using the *Löfgren C* alignment method. Results compare very well with the true ones obtained numerically and show that the tracking angle zeros should be at 70.025 and 115.985 mm, rather than at the 70.285 and 116.604 mm that all present *Löfgren C* protractors seem to be using[9].

7 ACKNOWLEDGMENT

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[9]Examples of protractors that use zeros at 70.29 and 116.60 mm for *Löfgren C* alignment are "Chpratz Protractor," "Stupid Protractor," and "Universal Stuff Protractor," which are all available at the Vinyl Engine Website https://www.vinylengine.com/cartridge-alignment -protractors.shtml; "Turntable Basics - Cartridge Alignment Tool" available at http://turntablebasics.com/align. html; "WallyTractor" available at https://www. wallyanalog.com/wallytractor; and "Protractor NG" available at http://www.feickert.org/index.php?id=17&L=1.

Additional thanks are also due to Mr. John Elison and Mr. Brian Kearns for spotting some mistakes in an early version of this text and sharing their insights into the events from 2000/2001 when the "Approximate" formulae for *Löfgren C* were published.

The author would also like to thank the anonymous reviewers who found additional errors in the submitted manuscript, at least one of which (in Fig. 1) would have been outright embarrassing.

During the review process, the author also learned that some master cutters existed in the 78-rpm era that used the pivoted arms [18]. Until further information becomes available, he believes that these were models used for amateur recordings before home tape recorders appeared. The vast majority of the commercial microgroove disks that have been mass-produced since the late 1940s seem to have been mastered on professional cutting lathes by Westrex, Scully, Presto, Neumann, Fairchild, Ortofon, etc., all of which had tangentially driven cutting styli.

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[10]A complete English translation of this patent will be available from the author by request.

THE AUTHOR

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As a student, Vladan Jovanovic worked as a rock journalist and editor of a Hi-Fi column in the Yugoslav rock maga-zine *Džuboks*. In 1982 he won the Belgrade Chamber of Commerce Award for his B.Sc. (Electrical Engineering) thesis on the geometry of the record player's tonearm.

After getting his Ph.D. from the University of Belgrade, Serbia, he worked in research and development for digital telecommunications. From 1991 to 1993 he was a Research Associate at the University of Toronto, Canada. From 1993 to 2000 he worked with mobile telephone operators in Canada and the United States. From 2000 to 2006 he was a Distinguished Member of Technical Staff at Lucent Tech-nologies, from 2006 to 2008 he was a CTO of Newfield Wireless, and he has worked as a private Consultant since.

Dr. Jovanovic authored over 30 technical papers in wire-less and digital communications and holds seven patents, one of which is for a tonearm adjustment protractor. Be-sides working as an expert witness for mobile telephony in the United States courts, in his retirement he has also written the book *Electronics of Rock and Roll*.